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The dipolar zero-modes of Einstein action: An informal summary with some new issues

Giovanni Modanese[1]

California Institute for Physics and Astrophysics
366 Cambridge Ave., Palo Alto, CA 94306

and

University of Bolzano – Industrial Engineering
Via Sernesi 1, 39100 Bolzano, Italy

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Abstract

We recall the main features of metric vacuum fluctuations which have the global property $\int d^4x\sqrt{g(x)}R(x) = 0$, even though $R(x) \neq 0$ locally. We stress that these fluctuations could mediate an anomalous coupling between the gravitational field and coherent matter. Some new issues are discussed: (1) these fluctuations still imply that $\langle T_{\mu\nu}(x)\rangle = 0$; (2) they are not extrema of the action; (3) for finite duration, their volume in phase space is not zero; (4) vacuum fluctuations of this kind are not allowed in QED; (5) their null-action property is a nonperturbative feature; (6) any *real* pure e.m. field generates zero-modes of this kind, too, up to terms of order G^2.

In this note we describe a set of gravitational field configurations, called "dipolar zero modes", which have not been considered earlier in the literature. They give an exactly null contribution to the pure Einstein action and can thus represent large vacuum fluctuations in the quantized theory of gravity.

The basic idea behind dipolar fluctuations was discussed for the first time in our earlier work on stability of Euclidean quantum gravity [1]; the Lorentzian case was treated in [2]. This year we made the first explicit computations and we were able to set some lower bounds on the strength of the fluctuations [3, 4]. Also we gave for the first time in Ref.s [3, 4]: (1) an estimate of possible suppression effects by cosmological or R^2-terms; (2) a computation of the total ADM energy of the zero modes; (3) a clarification (in the Lorentzian case) of the influence of matter fields on the fluctuations, with possible anomalous coupling.

Here, after a few general remarks about vacuum fluctuations and "spacetime foam" in quantum gravity, we shall set out the general features of the dipolar fluctuations (Section 1) and give some explicit order of magnitude calculations (Section 2). Then we shall show that a Λ-term cuts, to some extent, the dipolar fluctuations (Section 3); this can lead in certain cases to an anomalous coupling to matter (Section 4). In conclusion, we shall discuss a number of new topics not addressed in Ref.s [3, 4].

[1] e-mail address: giovanni.modanese@unibz.it

1 General features of the dipolar fluctuations

The functional integral of pure Einstein quantum gravity can be written as $z = \int d[g_{\mu\nu}] \exp(iS/\hbar)$, with $S = \int d^4x \sqrt{g(x)} R(x)$.

The "spacetime foam" [5] consists of fluctuations whose action does not exceed a quantity of order $\hbar$. This implies, for curvature fluctuations on a scale d, that $|R| < G/d^4$ (according to naive power counting) or $|R| < 1/(L_P d)$ (according to numerical lattice estimates [6]). Therefore, large fluctuations are expected to take place only at very small distances.

Since, however, the Einstein action is not positive definite, one can also expect some fluctuations due to peculiar cancellations of distinct contributions to the action, which are by themselves larger than $\hbar$.

In order to work this out explicitely, let us consider the Einstein equations with an *auxiliary* source $T_{\mu\nu}$:

$$R_{\mu\nu}(x) - \frac{1}{2} g_{\mu\nu}(x) R(x) = -8\pi G T_{\mu\nu}(x) \tag{1}$$

and their trace

$$R(x) = 8\pi G g^{\mu\nu}(x) T_{\mu\nu}(x) \equiv 8\pi G \mathrm{Tr} T(x) \tag{2}$$

Then consider a solution $g_{\mu\nu}(x)$ of (1) with any source satisfying the condition

$$\int d^4x \sqrt{g(x)} \mathrm{Tr} T(x) = 0 \tag{3}$$

In view of (2), this metric has zero action. We have constructed in this way a zero mode of the pure Einstein action. The source is unphysical, but it is "forgotten" after obtaining the metric. Condition (3) means in fact that it is a "dipolar" source, with a compensation between regions having positive and negative mass-energy density. Since this auxiliary source is used to construct a virtual field configuration, we shall sometimes call it a "virtual source".

2 Explicit computation to order G^2

We have then found the following "recipe" for constructing dipolar zero-modes of the pure Einstein action: given any source with zero integral (condition (3)), one solves the Einstein equations and finds the corresponding metric.

Note, however, that the condition on the source already contains $g_{\mu\nu}(x)$; furthermore, exact solutions are in general not known, and the approximation error could be such that the corresponding error on S is larger than $\hbar$. In this case we could still imagine that the zero-mode can be computed in principle, but we would not have in practice any definite idea of its properties. An explicit evaluation is therefore needed. To this end we consider static sources, with some free parameters (typically m_+, m_- and their sizes), in the weak field approximation. We have in this static case

$$S_{zero-mode} = -\frac{1}{2} \int d^4x \sqrt{g(x)} g^{\mu\nu}(x) T_{00}(x) \tag{4}$$

Using the Feynman propagator one finds to first order in G (compare [3])

$$h_{\mu\nu}(\mathbf{x}) = 2G(2\eta_{\mu 0}\eta_{\nu 0} - \eta_{\mu\nu}\eta_{00}) \int d^3y \frac{T^{00}(\mathbf{y})}{|\mathbf{x} - \mathbf{y}|} \tag{5}$$

It is straightforward to check from this expression that $\sqrt{g(x)}g^{00}(x) = 1 + o(G^2)$. This means that the action of the metric generated by a static source is simply

$$S = -\frac{1}{2}\int d^4xT_{00}(\mathbf{x}) + o(G^2) \tag{6}$$

and provided the integral of the mass-energy density vanishes, the field action is of order G^2, i.e. practically negligible, as shown in the following numerical example.

Consider a static dipolar source which is adiabatically switched on/off with a lifetime τ of the order of 1 s. Suppose that the spatial size r of the source is of the order of 1 cm and the two masses $m_\pm$ are of the order of 10^k g, i.e. 10^{37+k} cm^{-1} in natural units. Note that with this mass we have, for the ratio between the Schwarzschild radius and r, $r_{Schw.}/r \sim 10^{-29+k}$. This implies that we can compute $h_{\mu\nu}(x)$ in the weak field approximation, with negligible error. More precisely, the residual of second order in G is found to be

$$S^{order\ G^2}_{zero-mode} \sim \tau\frac{G^2m^2_\pm}{r^3} \sim 10^{-20+3k} \tag{7}$$

Therefore the field of a static virtual source of this magnitude order, satisfying the condition $\int d^3xT_{00}(\mathbf{x}) = 0$, has negligible action even if $k = 6$, corresponding to an apparent matter fluctuation with density 10^6 g/cm^3. This should be compared to the action of the corresponding "monopolar fluctuation", namely $S_{monopolar} = (1/2)\tau m + o(G^2) \sim 10^{47+k}$.

3 A Λ-term in the action cuts-off the fluctuations

The most recent estimates of the Hubble constant support a non-zero value of the cosmological constant of the order of $\Lambda \sim 10^{-50}$ cm^{-2}. The cosmological term in the gravitational action, to be added to the pure Einstein term, is

$$S_\Lambda = \frac{\Lambda}{8\pi G}\int d^4x\sqrt{g(x)} \tag{8}$$

It is possible to evaluate the contribution of the dipolar fluctuations to this term. This is easier for fluctuations with spherical symmetry, like those generated by virtual sources having the shape of "+/- shells" (compare [3]). In this case one can use the exact Schwarzschild metric outside the source and the spherically symmetric Newtonian field inside it. To leading order one finds that

$$\Delta S_\Lambda = \frac{\Lambda\tau}{8\pi G}\frac{1}{2}\int_{source} d^3x\mathrm{Tr}h(\mathbf{x}) =$$

$$= \frac{\Lambda\tau}{4\pi G}\int_{source} d^3xV_{Newt.}(\mathbf{x}) = \Lambda\tau mr^2Q \tag{9}$$

where Q is an adimensional factor which can be negative or positive, depending on the distribution of the positive and negative mass inside the virtual source.

Inserting for τ, m and r the same values as before, we find $\Delta S_\Lambda \sim 10^{-3+k}$. Remembering that the lower bound for pure gravity was $k \sim 6$, we see that the cosmological term cuts, to some extent, the dipolar fluctuations. This works even better for larger values of r.

4 Matter coupling vs. local changes in Λ

Let us consider now a scalar field ϕ coupled to gravity

$$L = \frac{1}{2}\left(\partial_\alpha\phi\partial^\alpha\phi - m^2\phi^2\right) \tag{10}$$

$$T_{\mu\nu} = \Pi_\mu\phi\partial_\nu\phi - g_{\mu\nu}L = \partial_\mu\partial_\nu\phi - g_{\mu\nu}L \tag{11}$$

$$S_{interaction} = \frac{1}{2}\int d^4x\sqrt{g(x)}T^{\mu\nu}(x)h_{\mu\nu}(x) \tag{12}$$

To lowest order in $h_{\mu\nu}$ the interaction action can be rewritten as

$$S_{interaction} = \frac{1}{2}\int d^4x\left(h_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - \mathrm{Tr}hL\right) \tag{13}$$

On the other hand, the cosmological action is, still to lowest order in $h_{\mu\nu}$ and expanding $\sqrt{g} = 1 + \frac{1}{2}\mathrm{Tr}h + ...$

$$S_\Lambda = \frac{\Lambda}{8\pi G}\int d^4x\left(1 + \frac{1}{2}\mathrm{Tr}h\right) \tag{14}$$

Therefore the sum of the two terms can be rewritten as

$$S_{interaction} + S_\Lambda = \frac{1}{2}\int d^4x h_{\mu\nu}\partial^\mu\phi\partial^\nu\phi + \frac{1}{2}\int d^4x\mathrm{Tr}h\left(\frac{\Lambda}{8\pi G} - L\right) \tag{15}$$

We see that to leading order the coupling of gravity to ϕ gives a typical source term $(h_{\mu\nu}\partial^\mu\phi\partial^\nu\phi)$ and subtracts from Λ the local density $8\pi GL(x)$. This separation is arbitrary, but useful and reasonable if the lagrangian density is such to affect locally the "natural" cosmological term and change the spectrum of gravitational vacuum fluctuations corresponding to virtual mass densities *much larger than the real density of* ϕ.

Let us give an example. Suppose that ϕ represents a coherent fluid with the density of ordinary matter ($\sim 1\ g/cm^3$). At the scale of 1 cm, with the observed value of Λ, the lower bound on virtual source density is $\sim 10^3\ g/cm^3$, which is much larger than the real density. If L is comparable to $\Lambda/8\pi G$ in some region, an inhomogeneity in the cut-off mechanism of the dipolar fluctuations will follow, and this effect could dominate the effects of the coupling $(h_{\mu\nu}\partial_\mu\phi\partial_\nu\phi)$ to real matter.

In our opinion, this dynamical mechanism could be the basis for an explanation of the weak gravitational modification by superconducting spinning disks which E. Podkletnov claimed to have observed under very special conditions [7, 8, 9]. A NASA/Argonne experimental team is at work to replicate the original results, which at this time are neither confirmed nor confuted.

From the very beginning of our theoretical analysis [10] we maintained that the disks described in Ref. [7] cannot be a source of gravitational field or perturbate it *in a classical sense*. This is because the gravitational coupling to matter is far too small, and neither the presence of Cooper pairs inside the disk nor its fast rotation help much under this respect. We have then been looking for an interaction process not constrained by the coupling, and a candidate for this are large quantum fluctuations (compare our phenomenological model in [11]).

More recently, anomalous gravity changes have been observed during a solar eclipse [12]. In this case the coherent matter which couples to the gravitational fluctuations could be the native iron which is abundant on the Moon.

5 Some remarks and new topics

(1) The fluctuations maintain $\langle T_{\mu\nu}(x)\rangle = 0$.

Even in the presence of strong quantum fluctuations, we expect that the vacuum average of $T_{\mu\nu}(x)$, computed through the functional integral, is zero. This is defined as the average of $\frac{1}{8\pi G}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)$. More physically, there must be no apparent mass-energy- momentum density, on the average, in the vacuum.

An analogous property holds in QED. Even in the presence of vacuum fluctuations, the vacuum average value of the four-current, defined through Maxwell's equations as $j_\mu = \partial^\nu F_{\mu\nu}$, vanishes.

In fact, some general properties of the functional integral ensure that the condition above on $T_{\mu\nu}$ is respected, without any need of restricting the integration space [13]. In other words, the fluctuations always average out in such a way to give a zero total virtual mass density at any point.

(2) The dipolar zero-modes have $S = 0$ but are not extrema of S.

The dipolar zero-modes are not a minimum of the pure Einstein action, because this would be equivalent of being a solution of the vacuum Einstein equations; but the dipolar zero-modes do not satisfy the Einstein equations in vacuum, even through they have the same action as the vacuum solutions ($S = 0$).

We can understand better the situation at hand through a bidimensional analogue (while, of course, the space of the possible metric configurations is infinite-dimensional). Let us consider the function $f(x, y) = x^2y^2$ and draw its cartesian plot. We find that it resembles a paraboloid, except for the fact that the axes $x = 0$ and $y = 0$ are a sort of "cuts"; along these axes the function takes the value $f = 0$, which is also the value at the origin. However, the origin is a minimum, while the points on the axes distinct from the origin are not minima.

(3) How much is the phase space volume of dipolar fluctuations?

The example above suggests another possible property of the dipolar zero-modes: in the same way as the "zero lines" of the function above have null measure with respect to its full bidimensional domain, one may think that the condition for the dipolar zero-modes defines a subspace of all field configurations having lower dimension and thus null measure. In this case the dipolar fluctuations would be suppressed because they have zero volume in phase space.

This would be true if the dipole condition $m_+ = m_-$ had to be satisfied exactly. In fact, however, we considered weaker conditions. We saw, for instance, that up to values of the virtual mass of the order of 10^6 g the difference between m_+ and m_-, i.e., the width of the "line in phase space", is of order τ^{-1}, where τ is the duration of the fluctuation.

The appearance of τ in this relation could also provide an upper bound on the duration of the fluctuations, making τ closer to the minimum duration allowed by the Heisenberg principle, like it happens for electromagnetic or scalar fields (compare [3, 4]). In other words, a long lifetime τ requires a very precise compensation between the positive and negative masses of the virtual dipole, thus implying a very small phase space volume for that configuration.

(4) Strong dipolar fluctuations are not allowed in QED.

This is an immediate consequence of the quadratic form of the electromagnetic lagrangian density: $L \propto (\mathbf{E}^2 - \mathbf{B}^2)$. The contribution to the action of the +/- charges in a virtual dipole is the same, therefore there are no cancellations. The dipolar fluctuations are not favoured with respect to the monopolar fluctuations.

The reasoning above also leads to the following statement.

(5) The null-action property of the dipolar fields is a non-perturbative feature.

In fact, suppose we limit ourselves to consider, as usual in perturbation theory, the part of the action quadratic in h in an expansion around the minimum. We then would be in the same situation as for the electromagnetic field. It is easy to see that the quadratic part of the gravitational action is not positive definite and has positive and negative eigenvalues in the Euclidean formulation, unlike the electromagnetic action [1]. Nevertheless, any single term would be the same for positive or negative virtual sources (see eq. (5)), without any cancellation.

In other words, the null action property of the dipolar zero modes cannot be obtained just considering the expansion in powers of h. This not because we are in strong field conditions, but because the zero-modes are not local minima of the action (compare Point 2).

(6) The $T_{\mu\nu}$ of any real pure electromagnetic field generates gravitational zero-modes, up to terms of order G^2.

Since the trace of the energy-momentum tensor of any electromagnetic field is zero, it is possible to use, instead of virtual unphysical dipolar mass sources, a *real* electromagnetic field as source of gravitational zero- modes. It needs to be a pure electromagnetic field in vacuum, thus any plane waves or wave packets are admissible. We stress again, however, that the real or virtual character of the source is irrelevant in order to obtain vacuum fluctuations.

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